

Lecture 1, part 2: ElmGossip

Gossip and Knowledge — ESSLLI 2025

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<https://malv.in/2025/esslli-gossip/>

Dynamic Gossip Graphs “by hand”

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Or we can *add* edges to the same drawing.

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What if we want to check many different call sequences?

This quickly becomes tedious. Hence, let's automate!

ElmGossip

Source code About

?

Gossip Protocols

$\sigma_k = \epsilon$

v

$\sigma_k = T, XZ$

+ Add constituent

Spider
?

Possible calls

X \hookleftarrow A

X \hookleftarrow Z

X \hookleftarrow Y

B \hookleftarrow X

B \hookleftarrow A

B \hookleftarrow Z

?

Call sequence

Execute

No call sequence entered

Call history

⏮
⏪
?

Gossip graph

Gossip graph input

Xyaz Axzy ZyAb BaZX Y

Examples

Canonical representation

Abce aBce BCde AbCD E

N
⏮
⏪

S
⏮
⏪

*

ElmGossip Source code About

Gossip Protocols ?

$\Omega_1 = \epsilon$

$\Omega_2 = \tau XZ$

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Possible calls

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Call sequence ?

Call sequence input

Execute

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Call history

* X \rightarrow A X \rightarrow A X \rightarrow A

X \rightarrow Z

Z \rightarrow X

Z \rightarrow A

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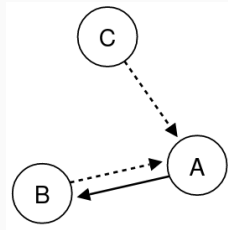
Ramon Meffert: *Tools for Gossip* (2021),
Bachelor thesis AI, University of Groningen.

Code: <https://github.com/RamonMeffert/elm-gossip>

Try it: <https://r3n.nl/elm-gossip/>

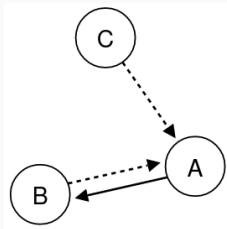
3

Short notation for gossip graphs



AB aB aC

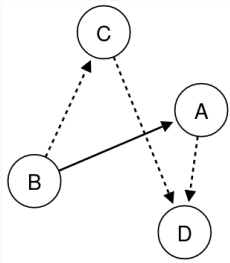
Short notation for gossip graphs



AB aB aC

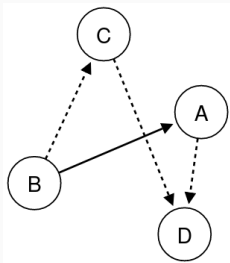
- A graph of n agents is described by n words separated by spaces.
- Knowing the **number** of agent a is denoted by a
- Knowing the **secret** of agent a is denoted by A

Examples

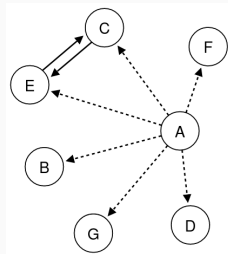


Ad ABc Cd D

Examples



Ad ABc Cd D



Abcdefg B CE D CE F G

Making calls

Click on a possible call to change the graph!

ElmGossip Source code About

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$\sigma_y = T \circ XZ$

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Possible calls

X ↪ A

X ↪ Z

X ↪ Y

B ↪ X

B ↪ A

B ↪ Z

Call sequence

Call sequence input

Execute

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X ↪ A

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Z ↪ X

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Gossip graph

Gossip graph input

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Canonical representation

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Protocols

In ElmGossip the following protocols are predefined:

Protocol	Calling condition
Any	\top
Call Once	$xy \notin \sigma_x \wedge yx \notin \sigma_x$
Lean New Secrets	$\neg S^\sigma xy$
Spider	$\sigma_x = \epsilon \vee \sigma_x = \tau; xz$
Token	$\sigma_x = \epsilon \vee \sigma_x = \tau; zx$
Weak Call Once	$xy \notin \sigma_x$

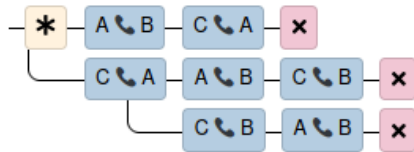
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And you can define your own custom protocols!

Call history



Comparing Protocols

Definition

We say that protocol A is *stronger* than protocol B iff the condition of A implies the condition of B . Hence, a *weaker* protocol can allow *more* calls!

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Lemma

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Lemma

- LNS is stronger than CO.
- CO is stronger than weak CO.
- All LNS sequences are also CO sequences. (But not vice versa \rightarrow exercise!)

Define your own protocol!

You can also define your own protocols in ElmGossip!

Example:

$$\sigma^x = \epsilon \vee xy \in \sigma^x$$

What does this say? 🤔

What ElmGossip does not cover

Hans also talked about the higher-order effects of gossip calls and K_i .

What would be a protocol condition that we **cannot** define in ElmGossip? 🤔

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Example: 🐷

$$PIG_{xy} := \hat{K}_x \exists z \neg(S_{xz} \leftrightarrow S_{yz})$$

Why can we not check such a protocol in ElmGossip?

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Why can we not check such a protocol in ElmGossip?

⇒ Tomorrow we will see a more general model checker for more general protocols.

Bonus: How does it work?

ElmGossip is written in the functional programming language *Elm*. Example piece of code:

```
containing : CallSequence -> AgentId -> CallSequence
containing sequence agent =
    case sequence of
        [] ->
            []
        call :: calls ->
            if includes call agent then
                call :: containing calls agent
            else
                containing calls agent
```

Links: <https://github.com/RamonMeffert/elm-gossip> · <https://guide.elm-lang.org/>

Exercises

1. With pen and paper, draw an initial total gossip graph for 4 agents, and execute the call sequence $ab; bc; cd; da$. Draw a new graph after each call, such that you have five graphs with four calls between them.
2. Open ElmGossip and check your drawings by executing the same call sequence there, step by step. Note that you first need to enter $Abcd$ $aBcd$ $abCd$ $abcD$ for the initial graph.
3. Is the sequence $ab; bc; cd; da$ allowed according to the ANY, the LNS and the CMO protocols?
4. Find a call sequence that is permitted for CMO but not for LNS.
5. Find a (non-total) gossip graph where CMO is weakly successful, but LNS is unsuccessful.
6. Define a protocol which allows more calls than LNS, but fewer than CMO.
7. Consider the “Spider” protocol. Why does its condition use $\sigma^x = \epsilon$ and not $\sigma = \epsilon$?