Gossip and Knowledge

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- Dynamic Gossip

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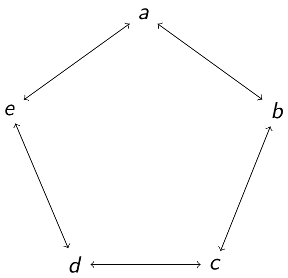
Gossip Graphs — dynamic gossip

We assumed that all agents can call all other agents.
The optimal schedule for a ‘ring’ takes $2n - 3$ calls.

Example 5 nodes: 6 calls is not possible but 7 calls is possible.

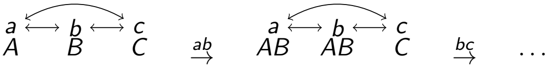
But if the agents can also exchange neighbours (numbers) 6 calls is possible again.



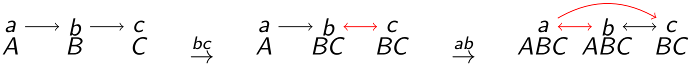
Dynamic Gossip — Learn New Secrets and Neighbours

Agents exchange all secrets and all numbers they know.

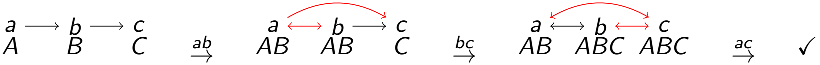
On fully connected graphs there is no difference.



On partially connected graphs LNS deadlock is possible. (After *bc*;*ab* agent *c* cannot call agent *a*, because *c* does not have *a*'s number.)



But sometimes LNS deadlock can be avoided.
(After *ab*;*bc* agent *a* calls agent *c*.)



When exactly? And how about other protocols?

Dynamic Gossip — Characterization of success

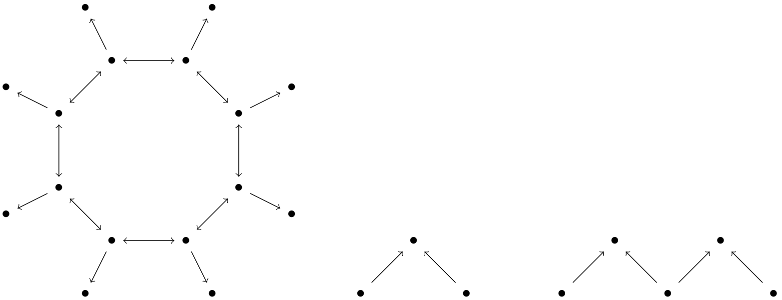
- A graph is weakly/strongly connected if there is an undirected/directed path between any two nodes.
- For gossip graphs: properties of the neighbour relation.
- No protocol is successful on a disconnected gossip graph.
- All presented protocols except LNS are (strongly or fairly) successful on weakly connected gossip graphs.

Relevant properties to show these results (let $G = (A, N, S)$):

- $S \subseteq N$
- $S^\sigma \subseteq N^\sigma$
- $S^\sigma \circ N \subseteq N^\sigma$ ($S^{\sigma.ab} \subseteq N^\sigma$)
- stable $\tau \subseteq \sigma^\infty$ satisfy $S^\tau_x = S^\tau_y$.
- *[in* TOK/SPI *every agent is a neighbour of a token holder]*

Dynamic Gossip — Characterization of LNS success

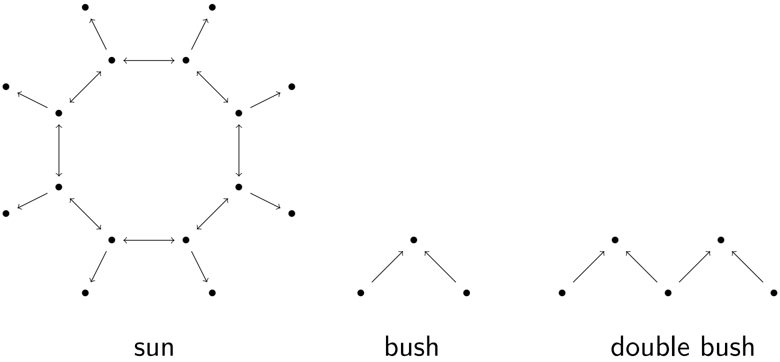
- If graph not weakly connected, unsuccessful. *Worse*:
  If graph a bush or double bush, unsuccessful.
- If graph strongly connected, strongly successful. *Better*:
  If graph a sun, strongly successful.



[vD, van Eijck, Pardo, Ramezanian, Schwarzentruber. *Dynamic Gossip*. Bulletin of the Iranian Mathematical Society, 2019]

Dynamic Gossip — Characterization of LNS success

- A sun is a strongly connected graph to which may be linked maximal nodes.
- A bush is a tree with branching factor in the root at least 2.
- A double bush consists of two bushes joined in a leaf linked to their roots.



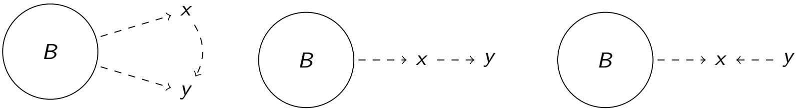
Dynamic Gossip — Characterization for LNS

- *LNS is (strongly) successful on a gossip graph*
  iff
  the gossip graph is a sun

Lemmas to obtain this result:

- if σ is LNS-maximal, then $S^\sigma = N^\sigma$.
  (otherwise there are a, b with $N^\sigma ab$ but $\neg S^\sigma ab$, so $\sigma.ab$ OK)
- if σ is LNS-maximal, then $S^\sigma \circ N^* = S^\sigma$
  (sketch: $S^\sigma \circ N \subseteq N^\sigma = S^\sigma$, iterate this ...).

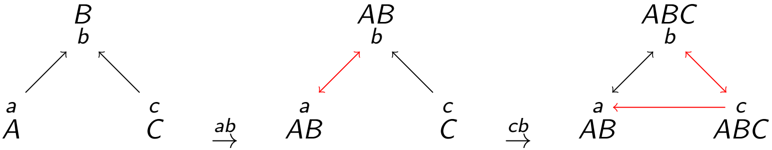
In all cases below there are LNS-maximal σ where y not expert:



Dynamic Gossip — Characterization for LNS

- *LNS is unsuccessful on a weakly connected gossip graph iff the weakly connected gossip graph is a bush or a double bush*

For example, LNS is unsuccessful on this bush:



The (rather) hard part of the proof is to construct a successful call sequence on an arbitrary weakly connected graph that is not a bush or a double bush.

[vD, van Eijck, Pardo, Ramezanian, Schwarzentruber. *Dynamic Gossip*. Bulletin of the Iranian Mathematical Society, 2019]

Dynamic Gossip — Characterization for LNS

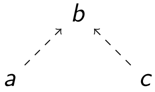
- *LNS is unsuccessful on a weakly connected gossip graph*
  iff
  the weakly connected gossip graph is a bush or a double bush

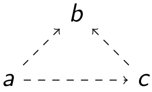
($\Leftarrow$) [the easy part] Given bush $G = (A, N, S)$ with root r, and LNS-permitted σ. Then:

1. $G|N^\sigma_x$ is a tree.
2. $N^\sigma_x \setminus S^\sigma_x = \begin{cases} \text{root of } G|N^\sigma_x & \text{if } \neg S^\sigma xr \\ \emptyset & \text{otherwise} \end{cases}$

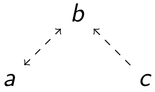
Sketch: Once the first child of the root calls the root, the subtrees generated by all other children are blocked (not LNS-permitted).

($\Leftarrow$) [the hard part, by example only, see BIMS 2019 for full proof]





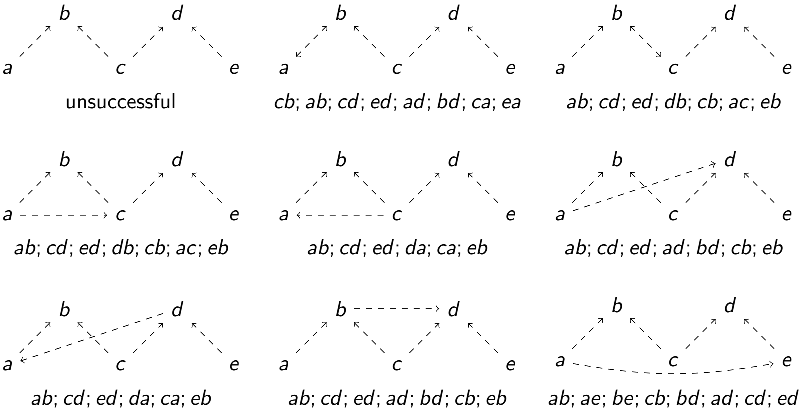
ab; *ac*; *bc*



cb; *ab*; *ca*

Dynamic Gossip — Characterization for LNS

Adding one edge to a double bush makes LNS successful . . .



All for today!