A Proof from 1988 that PDL has Interpolation?

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Advances in Modal Logic 2020

2020-08-26

15:00

Propositional Dynamic Logic (PDL)

Syntax

$$\begin{aligned}\phi \quad &::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid [a]\phi \\ a \quad &::= \quad A \mid a;a \mid a \cup a \mid a^* \mid \phi?\end{aligned}$$

Also expressible: "IF...THEN...ELSE..." and "WHILE...DO..."

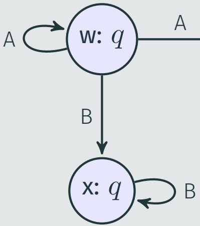
Syntax

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid [a]\phi$$

$$a \quad ::= \quad A \mid a; a \mid a \cup a \mid a^* \mid \phi?$$

Also expressible: "IF...THEN...ELSE..." and "WHILE...DO..."

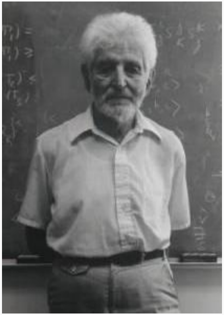
Example



$$\mathcal{M}, w \vDash \langle A; B\rangle q$$
$$\mathcal{M}, w \vDash [B]q$$
$$\mathcal{M}, w \vDash [B^*]q$$

$$\mathcal{M}, w \vDash \langle A\rangle(\langle A\rangle\neg q \wedge \langle B\rangle[B^*]q)$$

$L(\phi) := \{p \mid p \text{ occurs in } \phi\}$



William Craig

(1918 – 2016)

Definition

A logic has *Craig Interpolation* iff for any valid $\phi \to \psi$ there is an interpolant θ such that:

- $\phi \to \theta$ is valid
- $\theta \to \psi$ is valid
- $L(\theta) \subseteq L(\phi) \cap L(\psi)$

We then write $\phi \xrightarrow{\theta} \psi$.

Example

$(p \wedge q) \xrightarrow{q} (q \vee r)$

Logics that (we know) have Craig Interpolation

- Propositional Logic
- First-Order Logic
- Intuitionistic Logic
- Basic and Multi-modal logic (Madarász 1995)
- μ-Calculus (D'Agostino and Hollenberg 2000)

What about PDL?

Logics that (we know) have Craig Interpolation

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What about PDL?

Language of a formula := all atomic propositions *and programs*.

Example

$$[(A \cup B)^*](p \wedge q) \xrightarrow{[B^*]q} [(B; B)^*](q \vee [C]r)$$

Problem: How to find interpolants for $*$ systematically?

- Daniel Leivant: *Proof theoretic methodology for propositional dynamic logic*. Conference paper in LNCS, 1981.
- Manfred Borzechowski: *Tableau–Kalkül für PDL und Interpolation*. Diploma thesis, FU Berlin, 1988.
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- Marcus Kracht: Chapter *The open question* in *Tools and Techniques in Modal Logic*, 1999.

- It seems it was never really published.
- We now make an English translation available.
- Kracht (1999): not “possible to verify the argument”

<u>Definition.</u> Ein Tableau $\mathfrak{T}$ heißt *geschlossen*, wenn alle normalen freien Endknoten von $\mathfrak{T}$ geschlossen sind.

Im verbleibenden Teil von Abschnitt 1 zeigen wir, daß der hiermit definierte PDL-Kalkül vollständig ist; das heißt, daß jede Formelmenge $X \subseteq \mathfrak{F}$ genau dann erfüllbar ist, wenn kein geschlossenes Tableau für X existiert.

Doch zunächst geben wir ein Beispiel eines geschlossenen Tableaus für die Menge $X=\{\neg[(A\cup p?)^*]q, [A^*]q\}$.

$\neg[(A\cup p?)^*]q;\ [A^*]q$

(M+)

$\neg[(A\cup p?)^*]q^q;\ [A^*]q$

(n)

$\neg[(A\cup p?)^*]q^q;\ q;[A][A^*]q$

(¬n)

$\neg q;\ q;[A][A^*]q$ | $\neg[A\cup p?][(A\cup p?)^{(n)}]q^q;\ q;[A][A^*]q$

(¬u)

$\neg[A][(A\cup p?)^*]q^q;\ q;[A][A^*]q$ | $\neg[p?][(A\cup p?)^{(n)}]q^q;\ q;[A][A^*]q$

(At) | (¬?)

$\neg[(A\cup p?)^*]q^q;\ [A^*]q$ | $p;\neg[(A\cup p?)^{(n)}]q^q;\ q;[A][A^*]q$

Definition 16. *A tableau $\mathcal{T}$ is called* closed *when all normal free end nodes of $\mathcal{T}$ are closed.*

In the remainder of part 1 we show that the PDL-Calculus we defined in the previous sections is complete; that is, any set of formulas $X \subset \mathcal{F}$ is satisfiable if and only if there is no closed tableau for X.

However, before that we give an example of a closed tableau for the set $X = \{\neg[(A\cup p?)^*]q, [A^*]q\}$.

$\neg[(A\cup p?)^*]q\ ;\ [A^*]q$

$(M+)$

$\neg[(A\cup p?)^*]q^q\ ;\ [A^*]q$

(n)

$\neg[(A\cup p?)^*]q^q\ ;\ q\ ;\ [A][A^*]q$

$(\neg n)$ | $(\neg n)$

$\neg q\ ;\ q\ ;\ [A][A^*]q$ | $\neg[(A\cup p?)][(A\cup p?)^{(n)}]q^q\ ;\ q\ ;\ [A][A^*]q$

$(\neg\cup)$ | $(\neg\cup)$

$\neg[A][(A\cup p?)^*]q^q\ ;\ q\ ;\ [A][A^*]$ | $\neg[p?][(A\cup p?)^{(n)}]q\ ;\ q\ ;\ [A][A^*]$

(At) | $(\neg ?)$

$\neg[(A\cup p?)^*]q^q\ ;\ [A^*]$ | $p\ ;\ \neg[(A\cup p?)^{(n)}]q\ ;\ q\ ;\ [A][A^*]q^q$

Outline of the proof attempt

1. Define a tableaux system.
2. Show soundness and completeness.
3. Define interpolants for each node “bottom-up”.

Problems caused by $*$:

- How to ensure finite tableaux?
- How to define interpolants for $*$ steps?

classical rules:

$$(\neg)\ \frac{X; \neg\neg P}{X; P} \qquad (\wedge)\ \frac{X; P \wedge Q}{X; P; Q} \qquad (\neg\wedge)\ \frac{X; \neg(P \wedge Q)}{X; \neg P \ \mid\ X; \neg Q}$$

local rules:

$$(\neg\cup)\ \frac{X; \neg[a \cup b]P}{X; \neg[a]P \ \mid\ X; \neg[b]P} \qquad (\neg?)\ \frac{X; \neg[Q?]P}{X; Q; \neg P} \qquad (\neg;)\ \frac{X; \neg[a; b]P}{X; \neg[a][b]P}$$

$$(\cup)\ \frac{X; [a \cup b]P}{X; [a]P; [b]P} \qquad (?)\ \frac{X; [Q?]P}{x; \neg Q \ \mid\ X; P} \qquad (;)\ \frac{X; [a; b]P}{X; [a][b]P}$$

$$(\neg n)\ \frac{X; \neg[a^*]P}{X; \neg P \ \mid\ X; \neg[a][a^{(n)}]P} \qquad (n)\ \frac{X; [a^*]P}{X; P; [a][a^{(n)}]P}$$

Tableaux Rules: Part 2/2

PDL rules:

$$(M+)\ \frac{X; \neg[a_0] \dots [a_n]P}{X; \neg[a_0] \dots [a_n]P^P} \quad X \text{ free}$$

the *loading rule*,

$$(M-)\ \frac{X; \neg[a]P^R}{X; \neg[a]P}$$

the *liberation rule*,

$$(At)\ \frac{X; \neg[A]P^R}{X_A; \neg P^{R \setminus P}}$$

the *critical* rule.

marked rules:

$$(\neg\cup)\ \frac{X; \neg[a \cup b]P^R}{X; \neg[a]P^R \quad | \quad X; \neg[b]P^R} \qquad (\neg;)\ \frac{X; \neg[a;b]P^R}{X; \neg[a][b]P^R}$$

$$(\neg n)\ \frac{X; \neg[a^*]P^R}{X; \neg P^{R \setminus P} \quad | \quad X; \neg[a][a^{(n)}]P^R} \qquad (\neg ?)\ \frac{X; \neg[Q?]P^R}{X; Q; \neg P^{R \setminus P}}$$

where $(\dots)^{R \setminus P}$ indicates that R is removed iff $R = P$.

Tableaux Rules: Extra Conditions

1. On reaching $X; \neg[A]P$ or $X; [A]P$, change (n) back to $*$ in P.
2. Instead of $X; [a^{(n)}]P$ we always obtain X.
3. A rule must be applied to an n-formula whenever it is possible.
4. No rule may be applied to a $\neg[a^{(n)}]$-node.
5. To a node obtained using $(M+)$ we may not apply $(M-)$.
6. If a normal node t has a predecessor s with the same formulas and the path s...t uses (At) and is loaded if s is loaded, then t is an end node.
7. Every loaded node that is not an end note by 6 has a successor.

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A Full Proof Example

$$\neg[(A \cup p?)^*]q \; ; \; [A^*]q$$

$$(M+)$$

$$\neg[(A \cup p?)^*]q^q \; ; \; [A^*]q$$

$$(n)$$

$$\neg[(A \cup p?)^*]q^q \; ; \; q \; ; \; [A][A^*]q$$

$$(\neg n)1. \qquad\qquad (\neg n)$$

$$\neg q \; ; \; q \; ; \; [A][A^*]q \qquad\qquad \neg[(A \cup p?)][(A \cup p?)^{(n)}]q^q \; ; \; q \; ; \; [A][A^*]q$$

x

$$(\neg \cup) \qquad\qquad (\neg \cup)$$

$$\neg[A][(A \cup p?)^*]q^q \; ; \; q \; ; \; [A][A^*]q \qquad \neg[p?][(A \cup p?)^{(n)}]q^q \; ; \; q \; ; \; [A][A^*]q^q$$

$$(At) \qquad\qquad (\neg ?)$$

$$\neg[(A \cup p?)^*]q^q \; ; \; [A^*] \qquad\qquad p \; ; \; \neg[(A \cup p?)^{(n)}]q^q \; ; \; q \; ; \; [A][A^*]q$$

x x

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$$\neg[(A \cup p?)^*]q \,;\, [A^*]q$$

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$$\neg[(A \cup p?)^*]q^q \,;\, [A^*]q$$

(n)

$$\neg[(A \cup p?)^*]q^q \,;\, q \,;\, [A][A^*]q$$

$(\neg n)1.$ $\qquad$ $(\neg n)$

$$\neg q \,;\, q \,;\, [A][A^*]q \qquad \neg[(A \cup p?)][(A \cup p?)^{(n)}]q^q \,;\, q \,;\, [A][A^*]q$$

x
closed

$(\neg\cup)$ $\qquad$ $(\neg\cup)$

$$\neg[A][(A \cup p?)^*]q^q \,;\, q \,;\, [A][A^*]q \qquad \neg[p?][(A \cup p?)^{(n)}]q^q \,;\, q \,;\, [A][A^*]q^q$$

(At) $\qquad$ $(\neg ?)$

$$\neg[(A \cup p?)^*]q^q \,;\, [A^*] \qquad p \,;\, \neg[(A \cup p?)^{(n)}]q^q \,;\, q \,;\, [A][A^*]q$$

x $\qquad$ x

A Full Proof Example

$$\neg[(A \cup p?)^*]q \,;\, [A^*]q$$

$(M+)$

$$\neg[(A \cup p?)^*]q^q \,;\, [A^*]q$$

(n)

$$\neg[(A \cup p?)^*]q^q \,;\, q \,;\, [A][A^*]q$$

$(\neg n)1.$ $\qquad$ $(\neg n)$

$$\neg q \,;\, q \,;\, [A][A^*]q \qquad \neg[(A \cup p?)][(A \cup p?)^{(n)}]q^q \,;\, q \,;\, [A][A^*]q$$

x

$(\neg\cup)$ $\qquad$ $(\neg\cup)$

$$\neg[A][(A \cup p?)^*]q^q \,;\, q \,;\, [A][A^*]q \qquad \neg[p?][(A \cup p?)^{(n)}]q^q \,;\, q \,;\, [A][A^*]q^q$$

(At) $\qquad$ $(\neg ?)$

$$\neg[(A \cup p?)^*]q^q \,;\, [A^*] \qquad p \,;\, \neg[(A \cup p?)^{(n)}]q^q \,;\, q \,;\, [A][A^*]q$$

x $\qquad$ x

closed by condition 6

$$\neg[(A \cup p?)^*]q \, ; \, [A^*]q$$

$$(M+)$$

$$\neg[(A \cup p?)^*]q^q \, ; \, [A^*]q$$

$$(n)$$

$$\neg[(A \cup p?)^*]q^q \, ; \, q \, ; \, [A][A^*]q$$

$$(\neg n)1. \qquad\qquad (\neg n)$$

$$\neg q \, ; \, q \, ; \, [A][A^*]q \qquad\qquad \neg[(A \cup p?)][(A \cup p?)^{(n)}]q^q \, ; \, q \, ; \, [A][A^*]q$$

x

$$(\neg\cup) \qquad\qquad (\neg\cup)$$

$$\neg[A][(A \cup p?)^*]q^q \, ; \, q \, ; \, [A][A^*]q \qquad \neg[p?][(A \cup p?)^{(n)}]q^q \, ; \, q \, ; \, [A][A^*]q^q$$

$$(At) \qquad\qquad (\neg ?)$$

$$\neg[(A \cup p?)^*]q^q \, ; \, [A^*] \qquad\qquad p \, ; \, \neg[(A \cup p?)^{(n)}]q^q \, ; \, q \, ; \, [A][A^*]q$$

x

x

closed by condition 4

Definition

Write X_1/X_2 for a partitioned node $X = X_1 \cup X_2$ with $X_1 \cap X_2 = \emptyset$.

A formula θ is an interpolant for $X_1 \;/\; X_2$ iff

- θ is in $L(X_1) \cap L(X_2)$
- $X_1 \cup \{\neg\theta\}$ is inconsistent
- $\{\theta\} \cup X_2$ is inconsistent

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Corollary

A formula θ is an interpolant for $\{\phi\}/\{\neg\psi\}$ iff $\phi \xrightarrow{\theta} \psi$.

Definition

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- $\{\theta\} \cup X_2$ is inconsistent

Corollary

A formula θ is an interpolant for $\{\phi\}/\{\neg\psi\}$ iff $\phi \xrightarrow{\theta} \psi$.

Idea: Define interpolants depending on the rule application!

(Similar to Maehara's method for sequent calculi used by Leivant)

Consider the $(\neg\cup)$ rule, applied within X_2:

$$
\begin{array}{ccc}
& X_1 \ / \ X_2; \neg[a \cup b]P & \\
\swarrow_{(\neg\cup)} & & \searrow^{(\neg\cup)} \\
X_1 \ / \ X_2; \neg[a]P & & X_1 \ / \ X_2; \neg[b]P
\end{array}
$$

Tableaux Interpolation: Example

Consider the $(\neg\cup)$ rule, applied within X_2:

$$
\begin{array}{c}
\color{orange}{???} \\
X_1 \ / \ X_2; \neg[a \cup b]P
\end{array}
$$

$$
\begin{tikzcd}[column sep=small]
& X_1 \ / \ X_2; \neg[a \cup b]P \arrow[dl, dash, "(\neg\cup)"] \arrow[dr, dash, "(\neg\cup)"] & \\
\begin{array}{c} X_1 \ / \ X_2; \neg[a]P \\ \color{orange}{\theta_a} \end{array} & & \begin{array}{c} X_1 \ / \ X_2; \neg[b]P \\ \color{orange}{\theta_b} \end{array}
\end{tikzcd}
$$

Induction hypothesis provides θ_a and θ_b.

Tableaux Interpolation: Example

Consider the $(\neg\cup)$ rule, applied within X_2:

$$\begin{array}{ccc}
 & \begin{array}{c} \color{orange}{\theta_a \wedge \theta_b} \\ X_1 \;/\; X_2; \neg[a \cup b]P \end{array} & \\
\swarrow (\neg\cup) & & (\neg\cup) \searrow \\
\begin{array}{c} X_1 \;/\; X_2; \neg[a]P \\ \theta_a \end{array} & & \begin{array}{c} X_1 \;/\; X_2; \neg[b]P \\ \theta_b \end{array}
\end{array}$$

Induction hypothesis provides θ_a and θ_b.

We define $\theta := \theta_a \wedge \theta_b$.

A Full Interpolation Example

$$[A]p \wedge [B](p \wedge q) \ / \ \neg[A \cup B]p$$

$(\wedge)$

$$[A]p \,;\, [B](p \wedge q) \ / \ \neg[A \cup B]p$$

$(\neg\cup)$ $\quad$ $(\neg\cup)$

$$[A]p \,;\, [B](p \wedge q) \ / \ \neg[A]p \qquad\qquad [A]p \,;\, [B](p \wedge q) \ / \ \neg[B]p$$

x

(At)

$$(p \wedge q) \ / \ \neg p$$

$(\wedge)$

$$p \,;\, q \ / \ \neg p$$

x

A Full Interpolation Example

$$[A]p \wedge [B](p \wedge q) \ \ / \ \ \neg[A \cup B]p$$

$(\wedge)$

$$[A]p \,;\, [B](p \wedge q) \ \ / \ \ \neg[A \cup B]p$$

$(\neg\cup)$ $(\neg\cup)$

$$[A]p \,;\, [B](p \wedge q) \ \ / \ \ \neg[A]p \qquad\qquad [A]p \,;\, [B](p \wedge q) \ \ / \ \ \neg[B]p$$

x

(At)

$$(p \wedge q) \ \ / \ \ \neg p$$

$(\wedge)$

$$p \,;\, q \ \ / \ \ \neg p \qquad\qquad p$$

x

A Full Interpolation Example

$$[A]p \land [B](p \land q) \ / \ \neg[A \cup B]p$$

$(\land)$

$$[A]p \, ; [B](p \land q) \ / \ \neg[A \cup B]p$$

$(\neg\cup)$ $(\neg\cup)$

$$[A]p \, ; [B](p \land q) \ / \ \neg[A]p \qquad\qquad [A]p \, ; [B](p \land q) \ / \ \neg[B]p$$

x

(At)

$$(p \land q) \ / \ \neg p \qquad p$$

$(\land)$

$$p \, ; q \ / \ \neg p \qquad p$$

x

A Full Interpolation Example

$$[A]p \wedge [B](p \wedge q) \;\; / \;\; \neg[A \cup B]p$$

$(\wedge)$

$$[A]p \, ; [B](p \wedge q) \;\; / \;\; \neg[A \cup B]p$$

$(\neg\cup)$ $\quad$ $(\neg\cup)$

$$[A]p \, ; [B](p \wedge q) \;\; / \;\; \neg[A]p \qquad\qquad [A]p \, ; [B](p \wedge q) \;\; / \;\; \neg[B]p \qquad [B]p$$

x

(At)

$$(p \wedge q) \;\; / \;\; \neg p \qquad p$$

$(\wedge)$

$$p \, ; q \;\; / \;\; \neg p \qquad p$$

x

A Full Interpolation Example

$$[A]p \wedge [B](p \wedge q) \;/\; \neg[A \cup B]p$$

$(\wedge)$

$$[A]p \,;\, [B](p \wedge q) \;/\; \neg[A \cup B]p$$

$(\neg\cup)$ $\qquad$ $(\neg\cup)$

$$\color{orange}{[A]p}\ \color{black}{[A]p \,;\, [B](p \wedge q) \;/\; \neg[A]p} \qquad [A]p \,;\, [B](p \wedge q) \;/\; \neg[B]p \quad \color{orange}{[B]p}$$

x

(At)

$$(p \wedge q) \;/\; \neg p \qquad \color{orange}{p}$$

$(\wedge)$

$$p \,;\, q \;/\; \neg p \qquad \color{orange}{p}$$

x

A Full Interpolation Example

$$[A]p \wedge [B](p \wedge q) \;/\; \neg[A \cup B]p$$

$(\wedge)$

$$[A]p \,;\, [B](p \wedge q) \;/\; \neg[A \cup B]p \qquad [A]p \wedge [B]p$$

$(\neg\cup)$ $(\neg\cup)$

$$[A]p \quad [A]p \,;\, [B](p \wedge q) \;/\; \neg[A]p \qquad\qquad [A]p \,;\, [B](p \wedge q) \;/\; \neg[B]p \qquad [B]p$$

x

(At)

$$(p \wedge q) \;/\; \neg p \qquad p$$

$(\wedge)$

$$p \,;\, q \;/\; \neg p \qquad p$$

x

A Full Interpolation Example

$$[A]p \wedge [B](p \wedge q) \;\; / \;\; \neg[A \cup B]p \qquad [A]p \wedge [B]p$$

$(\wedge)$

$$[A]p \,;\, [B](p \wedge q) \;\; / \;\; \neg[A \cup B]p \qquad [A]p \wedge [B]p$$

$(\neg\cup)$ $\qquad$ $(\neg\cup)$

$$[A]p \quad [A]p \,;\, [B](p \wedge q) \;\; / \;\; \neg[A]p \qquad\qquad [A]p \,;\, [B](p \wedge q) \;\; / \;\; \neg[B]p \qquad [B]p$$

x

(At)

$$(p \wedge q) \;\; / \;\; \neg p \qquad p$$

$(\wedge)$

$$p \,;\, q \;\; / \;\; \neg p \qquad p$$

x

Conclusion

- 1988 attempt to show Craig Interpolation for PDL, now also in English.
- No obvious gap or problem found, so far.
- Many details about $*$ rules still to be checked!
- Question: Is condition 6 actually a circular system?
- Work in progress: A PDL prover with interpolation in Haskell.

If you are interested, please read, comment and help:

https://malv.in/2020/borzechowski-pdl

Bonus Slides

Craig Interpolation: Proof Methods

- Syntactic
  - propositional logic: replace unwanted atoms with $\top$ or $\bot$
  - no proof system needed
  - constructive
- Algebraic
  - amalgamation $\approx$ interpolation
  - not constructive
- Proof Theoretic
  - using sequent calculi or tableaux systems
  - start with a proof of the validity $\phi \to \psi$
  - construct interpolants for each step
  - sometimes constructive

Interpolation via Translation?

Wait, but we can translate PDL to the μ-Calculus, right?

- D'Agostino & Hollenberg: *Logical questions concerning the μ-Calculus: Interpolation, Lyndon and Łoś-Tarski*. JSL, 2000.

Yes, but …

0. Let $t : \mathcal{L}_{PDL} \to \mathcal{L}_\mu$ be the translation.
1. Suppose $\vDash_{PDL} \phi \to \psi$.
2. Then we have $\vDash_\mu t(\phi) \to t(\psi)$.
3. μ-Calculus has C-I, there is an interpolant $\gamma_\mu \in \mathcal{L}_\mu$:
   - $\vDash_\mu t(\phi) \to \gamma_m u$
   - $\vDash_\mu \gamma_m u \to t(\psi)$
   - $L(\gamma_\mu) = L(t(\phi)) \cap L(t(\psi))$
4. But now we still need $\gamma \in \mathcal{L}_{PDL}$ such that $t(\gamma) = \gamma_\mu$!?!?